PHYS 2220
Physics for Scientists and Engineers II
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Physics II Equation Sheet

Electrostatics

Electrostatics Fundamentals

Coulomb's Law $\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$

Electric Field $\mathbf{F} = q_0\mathbf{E}$

Gauss' Law $\Phi = \oiint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$

Electric Potential $U = q_0 V$

$$\mathbf{E} \to V: \quad \Delta V = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot d\boldsymbol{\ell} \qquad V \to \mathbf{E}: \quad \mathbf{E} = -\nabla V$$

Electric Fields and Potentials: Special Cases

Point Charge	$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$	$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
Point Charges	$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$	$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$
Dipole	$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}}{r^3}$	$V = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$
Circular Ring	$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{qz}{(z^2+R^2)^{3/2}}\hat{\mathbf{k}}$	$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2+z^2}}$
Circular Disc	$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{\mathbf{k}}$	$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$
Infinite Sheet	$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{k}}$	$V = -\frac{\sigma}{2\varepsilon_0}z + V_0$
Parallel Plate	$\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{k}}$	$V = -\frac{\sigma}{\varepsilon_0}z + V_0$
Infinite Line	$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$	$V = -\frac{\lambda}{2\pi\varepsilon_0}\ln(r) + V_0$
Conductors		
Inside	$\mathbf{E} = \mathbf{0}$	$V = V_0$
Surface	$\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$	

Charge Densities:

$$\lambda = \frac{dq}{dl}$$
$$\sigma = \frac{dq}{dA}$$
$$\rho = \frac{dq}{dV}$$

Electric Dipole:

$$\mathbf{p} = q\mathbf{d}$$

Potential Energy

Electric Dipole:

$$\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$$
$$U = -\mathbf{p}\cdot\mathbf{E}$$

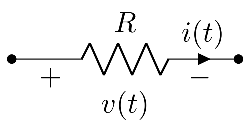
Charge Distribution:

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{(i,j)}\frac{q_i q_j}{r_{ij}}$$

Circuits: Components

Current: $i = \dfrac{dq}{dt}$ Current Density: $i = \iint \mathbf{J} \cdot d\mathbf{A}$ $i = AJ$

Resistors



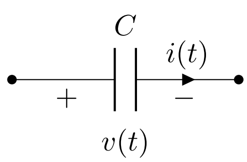
Resistance: $R = \rho \dfrac{L}{A}$ with $\mathbf{E} = \rho \mathbf{J}$

Voltage-Current: $v = Ri$ (Ohm's Law)

Power: $P = iv = iR^2 = \dfrac{v^2}{R}$

Series: $R_{eq} = \sum_i R_i$ Parallel: $\dfrac{1}{R_{eq}} = \sum_i \dfrac{1}{R_i}$

Capacitors



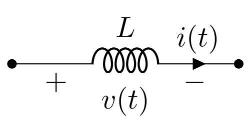
Capacitance: $C = \kappa \dfrac{\varepsilon_0 A}{d}$ (Parallel-Plate with Dielectric κ)

Voltage-Current: $i = C \dfrac{d}{dt} v$ $q = Cv$

Energy Stored: $U = \dfrac{1}{2} C v^2 = \dfrac{1}{2} \dfrac{q^2}{C}$

Series: $\dfrac{1}{C_{eq}} = \sum_i \dfrac{1}{C_i}$ Parallel: $C_{eq} = \sum_i C_i$

Inductors



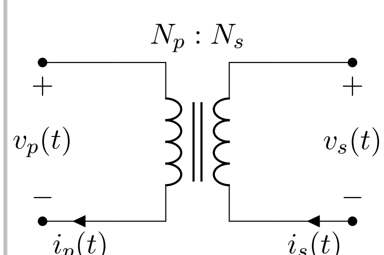
Inductance: $L = \dfrac{\mu_0 A N^2}{\ell}$ (Solenoid)

Voltage-Current: $v = L \dfrac{d}{dt} i$ $iL = N \Phi_B$

Energy Stored: $U = \dfrac{1}{2} L i^2$

Series: $L_{eq} = \sum_i L_i$ Parallel: $\dfrac{1}{L_{eq}} = \sum_i \dfrac{1}{L_i}$

Transformers



Voltage: $v_s = v_p \dfrac{N_s}{N_p}$

Current: $i_s = i_p \dfrac{N_p}{N_s}$

Power: $v_s i_s = v_p i_p$

Circuits: Theory

Fundamental Laws

Kirchhoff's Voltage Law

$$\sum_n v_n = 0$$

The sum of voltages around a closed loop is zero.

Kirchhoff's Current Law

$$\sum_n i_n = 0$$

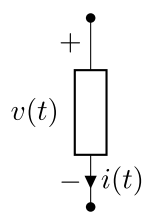
The sum of all currents leaving a node is zero.

Solving with KVL (Mesh Current)

1. Identify and label all mesh currents i_1, i_2, ...
2. Use Ohm's law as $v = iR$ to write one KVL for each loop.
3. Solve the system of equations.

Solving with KCL (Node Voltage)

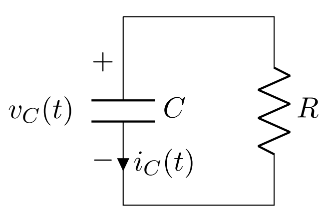
1. Identify and label all node voltages v_1, v_2, ...
2. Use Ohm's law as $i = \frac{v}{R}$ to write one KCL for each node.
3. Solve the system of equations.



Passive Sign Convention:
$v = v_+ - v_-$ and $+i$ flows from $+$ to $-$.

Power of a device: $P = iv$
$P < 0$ device supplies energy to circuit.
$P > 0$ device removes energy from the circuit.

RC Circuits

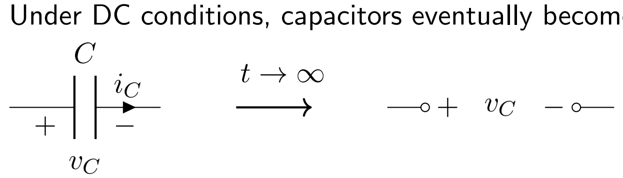


$$v_C(t) = v_C(\infty) + [v_C(0^-) - v_C(\infty)]e^{-t/\tau}, \quad t \geq 0$$

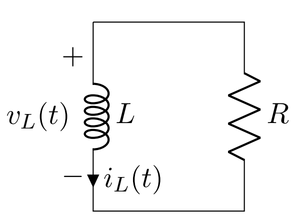
$$i_C(t) = -\frac{1}{R}[v_C(0^-) - v_C(\infty)]e^{-t/\tau}, \quad t \geq 0$$

$v_C(0^-)$	Voltage across capacitor before the circuit change.
$v_C(\infty)$	Voltage across the capacitor after the circuit change.
$\tau = RC$	Time constant.

Under DC conditions, capacitors eventually become open circuits.



RL Circuits

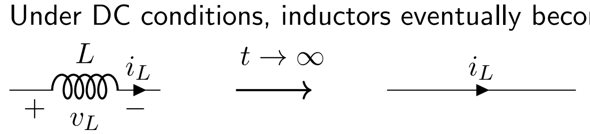


$$i_L(t) = i_L(\infty) + [i_L(0^-) - i_L(\infty)]e^{-t/\tau}, \quad t \geq 0$$

$$v_L(t) = -R[i_L(0^-) - i_L(\infty)]e^{-t/\tau}, \quad t \geq 0$$

$i_L(0^-)$	Current through inductor before the circuit change.
$i_L(\infty)$	Current through inductor after the circuit change.
$\tau = L/R$	Time constant.

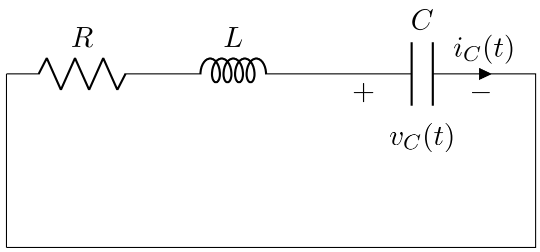
Under DC conditions, inductors eventually become short circuits.



Circuits: AC, RLC, and Oscillations

RLC Circuits

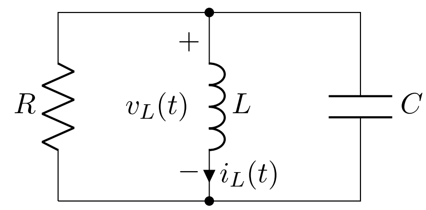
Series



$$0 = LC\frac{d^2}{dt^2}v_C(t) + RC\frac{d}{dt}v_C(t) + v_C(t)$$

$$\alpha = \frac{R}{2L} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Parallel

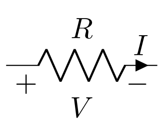


$$0 = LC\frac{d^2}{dt^2}i_L(t) + \frac{L}{R}\frac{d}{dt}i_L(t) + i_L(t)$$

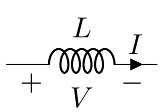
$$\alpha = \frac{1}{2RC} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Overdamped	Underdamped	Critically Damped
$\alpha > \omega_0$	$\alpha < \omega_0$	$\alpha = \omega_0$
$s_{12} = -\alpha \pm \omega_0$	$s_{12} = -\alpha \pm j\omega_0$	$s_{12} = -\alpha$
	$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	
$y(t) = A_1 e^{-s_1 t} + A_2 e^{-s_2 t}$	$y(t) = Ce^{-\alpha t}\cos(\omega_d t + \phi)$	$y(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$

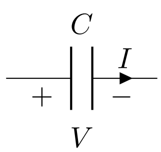
AC and Phasors



$V = RI \qquad Z_R = R$



$V = j\omega LI \qquad Z_L = j\omega L$



$j\omega CV = I \qquad Z_C = \frac{1}{j\omega C}$

AC and RMS Quantities

$$\begin{aligned} v(t) &= V_m \cos(\omega t + \phi) \\ &= \text{Re}\{V_m e^{j(\omega t + \phi)}\} \\ &\rightarrow V_m \angle \phi \ \ \text{(Phasor Notation)} \end{aligned}$$

RMS Value $\quad V_{rms} = \frac{V_m}{\sqrt{2}}$

AC Power $\quad P_{AC} = V_{rms} I_{rms} = \frac{V_m I_m}{2}$

Magnetism

Magnetism Fundamentals

Magnetic Flux	$\Phi_B = \iint \mathbf{B} \cdot d\mathbf{A}$	Magnetic Field	$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$
Net Magnetic Flux	$\oiint \mathbf{B} \cdot d\mathbf{A} = 0$	Force on a Wire	$\mathbf{F} = i\boldsymbol{\ell} \times \mathbf{B}$
Ampere's Law	$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 i_{enc}$	Torque on a Loop	$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$
Faraday's Law	$\oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt}\Phi_B$	Force on Two Wires	$F_{ab} = \frac{\mu_0 \ell i_a i_b}{2\pi r}$

Magnetic Fields: Special Cases

Biot-Savart Law	$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i d\boldsymbol{\ell} \times \hat{\mathbf{r}}}{r^2}$
Infinite Wire	$\mathbf{B} = \frac{\mu_0 i}{2\pi r}\hat{\boldsymbol{\ell}} \times \hat{\mathbf{r}}$
Magnetic Dipole	$\mathbf{B} = \frac{\mu_0}{2\pi}\frac{\boldsymbol{\mu}}{z^3}$
Circular Ring	$\mathbf{B} = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}\hat{\mathbf{k}}$
Solenoid	$B = \mu_0 i n$
Toroid	$B = \frac{\mu_0 i N}{2\pi r}$

Magnetic Dipole:

$$\boldsymbol{\mu} = Ni\mathbf{A}$$

Potential Energy:

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Turn Density:

$$n\ell = N$$

Induction

Lenz's Law: The magnetic field created by the induced current **opposes** the change in magnetic flux.

EMF in a Coil of N-Turns

$$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -N\frac{d}{dt}\Phi_B$$

Self-Induction

$$\mathcal{E} = -L\frac{di}{dt}$$

Mutual Induction

$$\mathcal{E}_2 = -M\frac{di_1}{dt}$$

$$\mathcal{E}_1 = -M\frac{di_2}{dt}$$

Electromagnetism

Maxwell's Equations

	Integral Form	Differential Form
Gauss's Law	$\oiint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss' Law for Magnetism	$\oiint \mathbf{B}\cdot d\mathbf{A} = 0$	$\nabla\cdot\mathbf{B} = 0$
Faraday-Lenz Law	$\oint \mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{d}{dt}\Phi_B$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Ampere-Maxwell Law	$\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 i_{enc} + \mu_0\varepsilon_0\frac{d}{dt}\Phi_E$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$

Electromagnetic Waves

Wave Equation: $\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = 0 \qquad \nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{B} = 0$

General Properties

Wavespeed	$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$
Wavelength	$\lambda = \frac{c}{f}$
Wavenumber	$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$
E- and B-Field	$E_m = B_m c$

1D Plane Wave	$E(x,t) = E_m\cos(kx - \omega t + \phi)$
Spherical Wave	$E(r,t) = \frac{E_m\cos(kr - \omega t + \phi)}{r}$
Poynting Vector	$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$
Intensity	$I = \frac{P_s}{A} = S_{avg} = \frac{E_{rms}^2}{c\mu_0} = \frac{E_m^2}{2c\mu_0}$
Polarization	$I = \frac{1}{2}I_0\cos^2\theta$

Optics

Reflection and Refraction

Reflection	$\theta_r = \theta_i$
Snell's Law	$n_1\sin\theta_1 = n_2\sin\theta_2$
Critical Angle	$\theta_c = \arcsin\left(\frac{n_1}{n_2}\right)$
Brewster's Angle	$\theta_b = \arctan\left(\frac{n_1}{n_2}\right)$

Mirrors and Thin Lenses

Postivie (+): Real Negative (-): Virtual

Planar Mirror	$i = -p$
Spherical Mirror & Lens	$\frac{1}{i} + \frac{1}{p} = \frac{1}{f}$
Focal Length	$f = \pm\frac{1}{2}r$
Lens Maker's Equation	$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$
Magnification	$\lvert m\rvert = \frac{\lvert i\rvert}{\lvert p\rvert} = \frac{h'}{h}$

Constants

Constant	Symbol	Value
Speed of light in a vacuum	c	$3.00 \times 10^8\,\mathrm{m/s}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Permittivity constant	ε_0	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Permeability constant	μ_0	$1.26 \times 10^{-6}\,\mathrm{H/m}$
Electron mass	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Proton mass	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Avogadro constant	N_A	$6.02 \times 10^{23}/\mathrm{mol}$