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Physics I Equation Sheet

Kinematics

1-D Kinematics

Position $x(t)$

Velocity $v(t) = \frac{d}{dt}x(t)$

Acceleration $a(t) = \frac{d}{dt}v(t)$

Special Case: Constant Acceleration

$a =$ constant $\qquad v(t) = v_0 + at$

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \qquad v^2 = v_0^2 + 2a(x - x_0)$

3-D Kinematics

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

$$\begin{aligned}\mathbf{v}(t) &= \frac{d}{dt}\mathbf{r}(t) = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k} \\ &= v_x(t)\mathbf{i} + v_y(t)\mathbf{j} + v_z\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}(t) &= \frac{d}{dt}\mathbf{v}(t) = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k} \\ &= a_x(t)\mathbf{i} + a_y(t)\mathbf{j} + a_z(t)\mathbf{k}\end{aligned}$$

$$\mathbf{r}(t) = \mathbf{r}_0 + \int_0^t \mathbf{v}(t')dt'$$

$$\mathbf{v}(t) = \mathbf{v}_0 + \int_0^t \mathbf{a}(t')dt'$$

Special Case: Projectile Motion

$a_x = 0 \qquad a_y = -g$

$v_x(t) = v_{0x} \qquad v_y(t) = v_{0y} - gt$

$x(t) = x_0 + v_{0x}t \qquad y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2$

Galilean Transformations

$$\mathbf{v} = \mathbf{w} + \mathbf{v}_F$$

$\mathbf{v}$: Velocity in stationary frame
$\mathbf{w}$: Velocity in moving frame
$\mathbf{v}_F$: Velocity of moving frame

Circular Motion

Motion on a Circle of Radius R

$$\mathbf{r}(t) = R\cos(\theta(t))\mathbf{i} + R\sin(\theta(t))\mathbf{j}$$

Angular Position $\theta(t)$

Angular Velocity $\omega(t) = \frac{d}{dt}(t)$

Angular Acceleration $\alpha(t) = \frac{d}{dt}\omega(t)$

Tangential Velocity $v_t = \omega R$

Tangential Acceleration $a_t = \alpha R$

Centripetal Acceleration $a_c = \omega^2 R = \frac{v_t^2}{R}$

Special Cases:
Uniform Circular Motion (Constant ω)
T: Period – time for one cycle
f: Frequency – cycles per second

$\alpha = 0 \implies a_t = 0 \qquad \omega =$ constant

$\omega = \frac{2\pi}{T} = 2\pi f \qquad v_t = \omega R$

Constant Angular Acceleration (Constant α)

$\alpha =$ constant $\qquad \omega(t) = \omega_0 + \alpha t$

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Dynamics

Newton's Three Laws

$\sum \mathbf{F} = \mathbf{0} \implies \mathbf{v} = \text{constant}$	Objects in motion tend to stay in motion unless acted upon by an external force.
$\sum \mathbf{F} = m\mathbf{a}$	The sum of all the forces acting on an object equals the object's mass times its acceleration.
$\mathbf{F}_{12} = -\mathbf{F}_{21}$	For every action there is an equal and opposite reaction.

Common Forces

Normal Force	$0 \leq N \leq \infty$	The normal force exerts enough force to keep an object from traveling through another.
Static Friction	$0 \leq F_S \leq \mu_s N$	Static friction exerts enough force to keep an object from sliding until the force required is greater than $\mu_s N$
Kinetic Friction	$F_K = \mu_k N$	Kinetic friction applies to a sliding object on a surface with friction and opposes the direction of motion.
Spring Force	$F_E = -k(x - x_0)$	Elastic forces, such as for a spring, act in the direction of the equilibrium position x_0 and are linearly proportional to the displacement from equilibrium $x - x_0$.

Circular Dynamics

For Circular Motion:

$$a_c = \omega^2 R = \frac{v_t^2}{R} \implies \sum F = F_c = m\omega^2 R = \frac{mv_t^2}{R}$$

The sum of the forces (net force) must equal F_c, the centripetal force, to have circular motion. The centripetal force and acceleration point towards the center of the circle.

Conservation Laws

Work & Kinetic Energy

Work – Variable Force	$\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r}$
Work – Constant Force	$\mathbf{F} \cdot \mathbf{d} = Fd\cos\theta$
Kinetic Energy	$K = \frac{1}{2}mv^2$
Power	$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$

Work-Energy Theorem

$$\Delta K = K_f - K_i = W_{net}$$

The change in kinetic energy of a system is equal to the net work done.

$W_{net} > 0$: Energy is transferred into the system
$W_{net} < 0$: Energy is transferred out of the system

Potential Energy

Conservative Force	$\oint \mathbf{F}_c \cdot d\mathbf{r} = 0$
Potential Energy	$\Delta U = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}_c \cdot d\mathbf{r}$
Force – Potential	$F_c(x) = -\frac{d}{dx}U(x)$
Gravitational Potential	$U_g(y) = mgy + U_0$
Elastic Potential	$U_e(x) = \frac{1}{2}k(x - x_0)^2$

Generalized Work-Energy Theorem

$$\Delta E = \Delta K + \Delta U = W_{ext}$$

The change in mechanical energy of a system is equal to the net work done by external (non-conservative) forces.

For an isolated system $W_{ext} = 0$ and $\Delta E = 0 \implies K_i + U_i = K_f + U_f$

Linear Momentum

Momentum	$\mathbf{p} = m\mathbf{v}$
Impluse	$\mathbf{J} = \Delta\mathbf{p} = \int_{t_1}^{t_2} \mathbf{F}(t)dt$
Newton's Second Law	$\mathbf{F} = \frac{d}{dt}\mathbf{p}$
Conservation of Momentum	$\sum \mathbf{p}_i = \sum \mathbf{p}_f$

Inelastic Collisions	Elastic Collisions
$\sum \mathbf{p}_i = \sum \mathbf{p}_f$	$\sum \mathbf{p}_i = \sum \mathbf{p}_f$ $K_i = K_f$

1D elastic collision with $v_{2i} = 0$:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} \quad v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i}$$

Angular Momentum

Angular Momentum Single Particle	$\mathbf{L} = \mathbf{r} \times \mathbf{p}$
Angular Momentum Rigid Body	$\mathbf{L} = I\boldsymbol{\omega}$
Newton's Second Law	$\boldsymbol{\tau} = \frac{d}{dt}\mathbf{L}$

Conservation of Angular Momentum:

$$\mathbf{L}_i = \mathbf{L}_f$$

if

$$\boldsymbol{\tau}_{ext} = \frac{d}{dt}\mathbf{L} = \mathbf{0}$$

Rigid Body Motion

Center of Mass

Center of Mass	$\mathbf{r}_{CM} = \frac{\sum_i m_i \mathbf{r}_i}{M}$ $M = \sum_i m_i$	The center of mass represents the point about which a rigid body behaves as a point mass with total mass M.
Linear Momentum	$\mathbf{p} = M\mathbf{v}_{CM}$	The total linear momentum of a rigid body is the total mass times the velocity of the center of mass.
Newton's 2nd Law	$\sum \mathbf{F} = M\mathbf{a}_{CM}$	The sum of all forces causes a linear acceleration of the center of mass.

Moment of Inertia

Moment of Inertia	$I = \sum_i m_i \mathbf{r}_i^2$	The moment of inertia represent how a rigid body resists rotation about its center of mass.
Torque	$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ $\tau = rF\sin\phi$	A force applied at a distance from a rotation point induces a torque about the axis of rotation.
Newton's 2nd Law	$\sum_i \boldsymbol{\tau}_i = I\boldsymbol{\alpha}$	The sum of all torques causes an angular acceleration about the center of mass or rotation point.

Energy of a Rigid Body

Translational Kinetic Energy	$K_t = \frac{1}{2}Mv_{CM}^2$
Rotational Kinetic Energy	$K_r = \frac{1}{2}I\omega^2$
Total Kinetic Energy	$K = K_t + K_r$

Rolling Without Slipping:

$$v_{CM} = R\omega \qquad a_{CM} = R\alpha$$

$$K = K_t + K_r = \frac{1}{2}v_{CM}^2\left(M + \frac{I}{R^2}\right)$$

Waves and Oscillations

Simple Harmonic Oscillator

General Solution

Displacement $x(t) = A\cos(\omega_0 t + \phi)$

Velocity $v(t) = -A\omega_0 \sin(\omega_0 t + \phi)$

Acceleration $a(t) = -A\omega_0^2 \cos(\omega_0 t + \phi)$

Resonance Frequency

Mass-Spring $\omega_0 = \sqrt{\dfrac{k}{m}}$

Pendulum $\omega_0 = \sqrt{\dfrac{g}{L}}$

Properties

Period $T = \dfrac{2\pi}{\omega_0}$

Frequency $f = \dfrac{\omega_0}{2\pi} = \dfrac{1}{T}$

Restoring Force $F(t) = -Cx(t)$

Kinetic Energy $K(t) = \dfrac{A^2\omega_0^2 m}{2}\sin^2(\omega_0 t + \phi)$

Potential Energy $U(t) = \dfrac{A^2\omega_0^2 m}{2}\cos^2(\omega_0 t + \phi)$

Damped Oscillator: $\omega_d = \sqrt{|\alpha^2 - \omega_0^2|}$

Overdamped: $\alpha > \omega_0$

$$x(t) = e^{-\alpha t}\left[A\cosh(\omega_d t) + B\sinh(\omega_d t)\right]$$

Underdamped: $\alpha < \omega_0$

$$x(t) = e^{-\alpha t}\left[A\cos(\omega_d t) + B\sin(\omega_d t)\right]$$

Waves

General Properties

Wavespeed c

Wavelength $\lambda = \dfrac{c}{f}$

Wavenumber $k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{c}$

Traveling Waves

D'Alembert Solution $y(x,t) = f(x - ct) + g(x + ct)$

Sinusoidal Solution $y(x,t) = A\cos(kx \pm \omega t + \phi)$

Standing Waves

Fixed-Fixed String $y_n(x,t) = A_n \sin(k_n x)\cos(\omega_n t + \phi)$

$$k_n = \frac{n\pi}{L}, \quad n = 1, 2, 3, \cdots$$

Acoustics

1D Plane Wave $p(x,t) = A\cos(kx - \omega t + \phi)$

Spherical Wave $p(r,t) = \dfrac{A\cos(kx - \omega t + \phi)}{r}$

Sound Intensity $I = \dfrac{P}{S}$

(dB) $L = 10\log_{10} I/I_0$

$I_0 = 1\,\mathrm{pW}$

Pipe Resonance Frequencies

Open-Open $p_n(x,t) = A_n \sin(k_n x)\cos(\omega_n)$

$k_n = \dfrac{n\pi}{L}, \quad n = 1, 2, 3, \cdots$

Closed-Closed $p_n(x,t) = A_n \cos(k_n x)\cos(\omega_n)$

$k_n = \dfrac{n\pi}{L}, \quad n = 1, 2, 3, \cdots$

Open-Closed $p_n(x,t) = A_n \cos(k_n x)\cos(\omega_n)$

$k_n = \dfrac{n\pi}{4L}, \quad n = 1, 3, 5, \cdots$