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꽹과리의 모달 해석 및 베셀 급수 전개

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Modal Analysis and Bessel Series Expansion of a Kkwaenggwari

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요약

Understanding the radiation characteristics of musical instruments is important in many audio applications such as room acoustic design [1], physical modeling synthesis [2], and microphone placement [3]. However, most studies on musical instruments focus on either the modal behavior or the directivity characteristics as isolated systems, neglecting how vibrational behavior influences the radiated sound. This study aims to improve understanding between the relationship between the modal response of the kkwaenggwari and its radiated field by developing a method to project a discretely measured vibrational response onto a continuous modal basis with a known radiation pattern. In this way, the directivity pattern of any vibrational distribution may be directly calculated through a modal expansion.

The vibrational data of the instrument followed from a Scanning Laser Doppler Vibrometery (SLDV) scan. The scan incorporated 224 discrete points across the instrument's front face suspended using approximately free boundary conditions. A loudspeaker excited the instrument using white noise to produce frequency response measurements between the reference noise signal and the surface velocity. Figure 1 shows the measurement set-up for the SLDV data collection.

Figure 1: SLDV measurement set-up.

While the SLDV measurement provides information about the vibrational behavior of the instrument, the data is only available at a discrete set of points. However, predicting the radiated acoustic pressure requires a continuously defined representation to apply theoretical models. A least-squares fit between the discretely measured data and a Bessel series expansion provides a means to continuously interpolate the data over a circular region using a complete basis set. The model thus assumes that the velocity may be expanded as

$$u(r,\phi) = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} A_{nm} J_n\left(\frac{\alpha_{nm} r}{R}\right) e^{in\phi} \tag{1}$$

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where $u(r,\phi)$ is the surface velocity at radius r and angle ϕ, J_n are the Bessel functions of the first kind of order n and α_{nm} represent the mth zero of the nth order Bessel function, R is the radius of the instrument, and A_{nm} are the expansion coefficients. In practice, the infinite sum must be truncated to a finite order N and maximum zero number M. Evaluating Eq. (1) at each of the 224 discretely sampled points provides a system of equations which may be solved using a least-squares fit to calculate the expansion coefficients. A critical aspect of the continuous modal reconstruction is the frequency dependent selection of the proper maximum basis function orders and zeros to avoid spatial overfitting. Figure 2 shows results for eight different modal peaks appearing in the SLDV data. In each case, the maximum Bessel function order N and maximum zero M appear in parenthesis. Overall, the results demonstrate that even with low values of N and M, accurate reconstruction of the velocity distribution is possible, enabling future directivity projections of the radiated acoustic field.

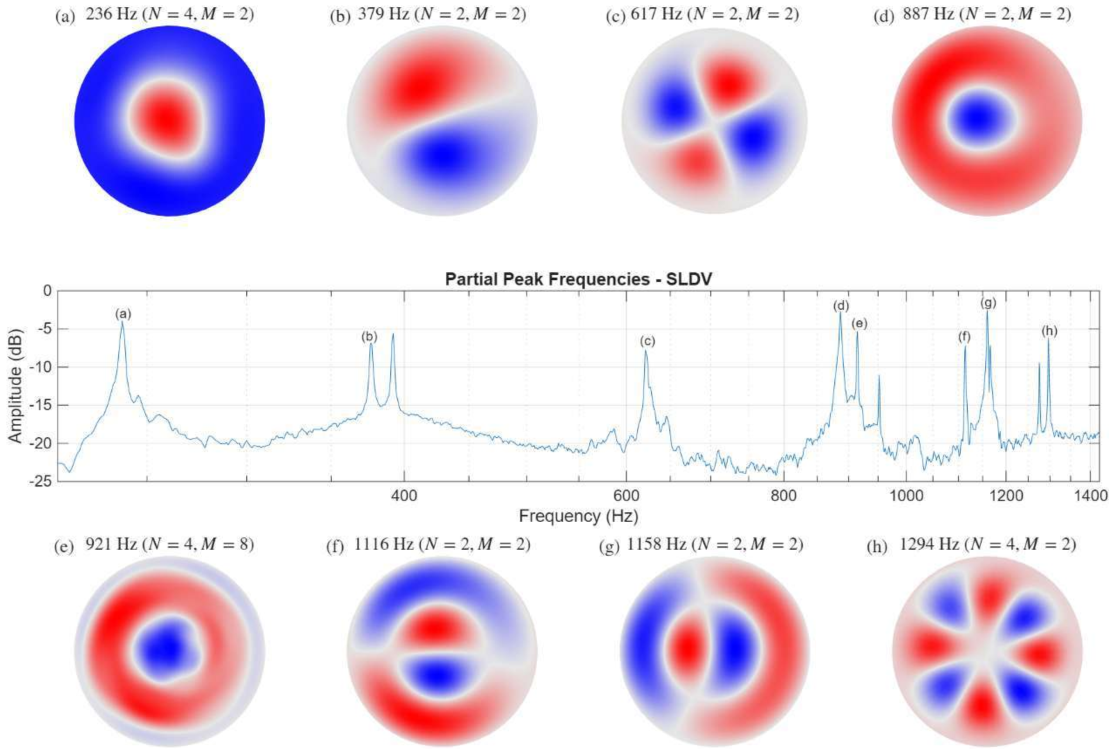

Figure 2. Reconstructed velocity over the scanning surface using the Bessel series expansion for select modes of the instrument.

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